From Operator Algebra to Noncommutative Function Theory

Paul S. Muhly
University of Iowa

Abstract

In the summer of 1966, when I was taking a reading course from Paul Halmos, he told me: "If you want to study a question about operators on infinite dimensional Hilbert spaces, first formulate it in the setting of finite dimensional spaces. Answer it there, and only then move on to the infinite dimensional setting." While this admonition may seem naive, I want to show how taking it seriously can reveal interesting connections between operator algebra and the theory of analytic functions of noncommutative variables. Very briefly: The journey begins with the work of Murray and von Neumann on rings of operators. Much of their inspiration came from finite group representation theory, and the algebras they constructed are viewed by many as infinite dimensional versions of semisimple algebras. When trying to fit non-semisimple algebras into operator algebra, Baruch Solel and I were inspired by algebraic theories developed in the late 40s and were led to think about tensor algebras and the theory of quivers (i.e., finite directed graphs). The algebras we constructed could profitably be studied as spaces of analytic functions on the algebras' representations. These are the free functions to which the title refers. The presentation will be largely historical and non-technical. It will require background only from first-year graduate courses in algebra and analysis.